Lecture 3 - Jan 27

Continuous variables have an infinite number of possible values, so they need a probability density.

$$\mathbb{P}[a < X < b] = \int_a^b f(x)dx \tag{1}$$

the probability density at x is $f(x)$ and $0 \leq f(x)$. Note that $f(x)$ can be arbitrarily large. The area under a probability density function is always 1.

The cumulative probability or cumulative density function, $F(x)$ gives the probability of $X \leq x$ for any x.

$$\mathbb{P}[X < b] = F(x) = \int_{\min[x]}^b f(x)dx \tag{2}$$

for many distributions, e.g. the normal, $\min[x] = -\infty$. If you have an analytical solution for $F(x)$, then:

$$\begin{aligned} \mathbb{P}[a < X < b] &= \mathbb{P}(x < b) - \mathbb{P}(x < a) && (3) \\ &= F(b) - F(a) && (4) \end{aligned}$$

(discussion of bird data: movement events and times).

For an exponential distribution:

$$f(t) = \beta e^{-\beta t} \tag{5}$$

$0 \leq t$ and $\beta > 0$. It is the simplest model of waiting times because the rate of events is constant. If that holds, then the waiting times will follow an exponential distribution and the mean waiting time will be $1/\beta$.

If we have a set of waiting times ($T = [t_1, t_2, \ldots t_n]$), and we can assume that they are independent, then the likelihood can be produced by taking the product of the densities for each waiting time:

$$\begin{aligned} L(\beta) = f(T \mid \beta) &= \prod_{i=1}^n \beta e^{-\beta t_i} && (6) \\ \ell(\beta) = \ln L(\beta) = \ln f(T \mid \beta) = &= \sum_{i=1}^n [\ln \beta - \beta t_i] && (7) \\ &= n \ln[\beta] - n\beta\bar{t} && (8) \\ \frac{\partial \ell(\beta)}{\partial \beta} = \frac{n}{\beta} - n\bar{t} & && (9) \\ \frac{n}{\hat{\beta}} - n\bar{t} &= 0 && (10) \\ \hat{\beta} = 1/\bar{t} & && (11) \end{aligned}$$