Hastings ratio $= \frac{\mathbb{P}\left(\text{proposing } \theta^\star \rightarrow \theta\right)}{\mathbb{P}\left(\text{proposing } \theta \rightarrow \theta^\star\right)} = \frac{g\left(\boldsymbol{u}^\star\right)}{g\left(\boldsymbol{u}\right)}|J|$

Peter Green's recipe:

1. draw a vector of k random variates, $\boldsymbol{u}$, from a prob. distribution, $g(\boldsymbol{u})$
2. Use a deterministic function, h, to "map" the current parameters, $\boldsymbol{\theta}$, and $\boldsymbol{u}$ to a the proposed vector of parameters: $\boldsymbol{\theta}^\star = h(\boldsymbol{\theta}, \boldsymbol{u})$
3. Consider the reverse move, and express the random variates, $\boldsymbol{u}^\star$, required in the reverse the proposal.
4. Express functions for the elements of $\{\boldsymbol{\theta}^\star, \boldsymbol{u}^\star\}$ as a function of $\{\boldsymbol{\theta}, \boldsymbol{u}\}$
5. Calculate the absolute value of the determinant of the Jacobian matrix:

$$J = \left[\begin{array}{cccccc} \partial\theta_1^\star/\partial\theta_1 & \partial\theta_1^\star/\partial\theta_2 & \dots & \partial\theta_1^\star/\partial u_1 & \dots & \partial\theta_1^\star/\partial u_k \\ \partial\theta_2^\star/\partial\theta_1 & \partial\theta_2^\star/\partial\theta_2 & \dots & \partial\theta_2^\star/\partial u_1 & \dots & \partial\theta_2^\star/\partial u_k \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \partial u_1^\star/\partial\theta_1 & \partial u_1^\star/\partial\theta_2 & \dots & \partial u_1^\star/\partial u_1 & \dots & \partial u_1^\star/\partial u_k \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \partial u_k^\star/\partial\theta_1 & \partial u_k^\star/\partial\theta_2 & \dots & \partial u_k^\star/\partial u_1 & \dots & \partial u_k^\star/\partial u_k \end{array}\right]$$

Sliding window move with window width λ:

$$
\begin{aligned}
u \text{ and } u^\star &\sim \text{Uniform}[0,1] \\
g(u) &= g(u^\star) = 1 \\
\theta^\star &= h(\theta, u) \\
&= \theta + \lambda(u - .5) \\
\theta &= h(\theta^\star, u^\star) \\
&= \theta^\star + \lambda(u^\star - .5) \\
&= \theta + \lambda(u - .5) + \lambda(u^\star - .5) \\
(\theta - \theta) = 0 &= \lambda(u + u^\star - 1) \\
u^\star &= 1 - u \\
J &= \begin{bmatrix} 1 & \lambda \\ 0 & -1 \end{bmatrix} \\
|J| &= 1 \\
\text{Hastings ratio} &= \frac{1}{1}(1) = 1
\end{aligned}
$$

Scaler window move:

$$\begin{aligned}
u \text{ and } u^\star &\sim \text{Uniform}[0,1] \\
g(u) &= g(u^\star) = 1 \\
\theta^\star &= \theta e^{\lambda(u-.5)} \\
\theta &= \theta^\star e^{\lambda(u^\star-.5)} \\
&= \theta e^{\lambda(u-.5)} e^{\lambda(u^\star-.5)} \\
1 &= e^{\lambda(u+u^\star-1)} \\
0 &= \lambda(u+u^\star-1) \\
u^\star &= 1-u
\end{aligned}$$

Scaler window move:

$$
\begin{aligned}
u \text{ and } u^\star &\sim \text{Uniform}[0,1] \\
g(u) &= g(u^\star) = 1 \\
\theta^\star &= \theta e^{\lambda(u-.5)} \\
\theta &= \theta^\star e^{\lambda(u^\star-.5)} \\
&= \theta e^{\lambda(u-.5)} e^{\lambda(u^\star-.5)} \\
1 &= e^{\lambda(u+u^\star-1)} \\
0 &= \lambda(u+u^\star-1) \\
u^\star &= 1-u \\
J &= \begin{bmatrix} e^{\lambda(u-.5)} & \lambda\theta e^{\lambda(u-.5)} \\ 0 & -1 \end{bmatrix} \\
|J| &= e^{\lambda(u-.5)} \\
\text{Hastings ratio} &= \frac{1}{1}(e^{\lambda(u-.5)}) = e^{\lambda(u-.5)} = \frac{\theta^\star}{\theta}
\end{aligned}
$$