GLM notes of JKK's lecture

I am using I for the incidence matrix, while John used X in lecture.

$$\ell = C - \frac{1}{2}\ln|V| - \frac{1}{2}(Y - I\eta)^T V^{-1}(Y - I\eta) \tag{1}$$

$$Y = \begin{bmatrix} y_{11} \\ y_{12} \\ y_{21} \\ y_{22} \\ y_{31} \\ y_{32} \end{bmatrix} \tag{2}$$

$$\eta = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} \tag{3}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$\mathbb{E}[Y] = I\eta = \begin{bmatrix} \mu_1 \\ \mu_1 \\ \mu_2 \\ \mu_2 \\ \mu_3 \\ \mu_3 \end{bmatrix} \tag{5}$$

$$\tag{6}$$

		Diet	
		C	T
Fam	1	y_{C11}	y_{T11}
		y_{C12}	y_{T12}
	2	y_{C21}	y_{T21}
		y_{C22}	y_{T22}

$$Y_{ijk} = \mu + \tau_i + F_j + C_{ij} + \epsilon_{ijk} \tag{7}$$

$$Y = \begin{bmatrix} y_{C11} \\ y_{C12} \\ y_{C21} \\ y_{C22} \\ y_{T11} \\ y_{T12} \\ y_{T21} \\ y_{T22} \end{bmatrix} \tag{8}$$

$$\eta = \begin{bmatrix} \mu \\ \tau_C \\ \tau_T \end{bmatrix} \tag{9}$$

$$\text{Var}[Y_{C11}] = \nu = \sigma_F^2 + \sigma_C^2 + \sigma_E^2 \tag{10}$$

$$\text{Cov}[Y_{C11}, Y_{C12}] = c_1 = = \text{Cov}[F_1 + C_{C1} + \epsilon_{C11}, F_1 + C_{C1} + \epsilon_{C12}] \tag{11}$$

$$= \sigma_F^2 + \sigma_C^2 \tag{12}$$

$$\text{Cov}[Y_{T11}, Y_{T12}] = \text{Cov}[F_1 + C_{T1} + \epsilon_{T11}, F_1 + C_{T1} + \epsilon_{T12}] \tag{13}$$

$$= \sigma_F^2 + \sigma_C^2 = c_1 \tag{14}$$

$$\text{Cov}[Y_{C11}, Y_{C21}] = = \text{Cov}[F_1 + C_{C1} + \epsilon_{C11}, F_2 + C_{T2} + \epsilon_{C22}] \tag{15}$$

$$= 0 \tag{16}$$

$$\text{Cov}[Y_{C11}, Y_{T11}] = = \text{Cov}[F_1 + C_{C1} + \epsilon_{C11}, F_1 + C_{T1} + \epsilon_{T12}] \tag{17}$$

$$= \sigma_F^2 \tag{18}$$

$$V = \begin{bmatrix} \nu & c_1 & 0 & 0 & \sigma_F^2 & \sigma_F^2 & 0 & 0 \\ c_1 & \nu & 0 & 0 & \sigma_F^2 & \sigma_F^2 & 0 & 0 \\ 0 & 0 & \nu & c_1 & 0 & 0 & \sigma_F^2 & \sigma_F^2 \\ 0 & 0 & c_1 & \nu & 0 & 0 & \sigma_F^2 & \sigma_F^2 \\ \sigma_F^2 & \sigma_F^2 & 0 & 0 & \nu & c_1 & 0 & 0 \\ \sigma_F^2 & \sigma_F^2 & 0 & 0 & c_1 & \nu & 0 & 0 \\ 0 & 0 & \sigma_F^2 & \sigma_F^2 & 0 & 0 & \nu & c_1 \\ 0 & 0 & \sigma_F^2 & \sigma_F^2 & 0 & 0 & c_1 & \nu \end{bmatrix} \tag{19}$$

Reordering the order of data could give us a block diagonal. Which is nice because matrix inversion is $\mathcal{O}(N^3)$:

$$Y = \begin{bmatrix} y_{C11} \\ y_{C12} \\ y_{T11} \\ y_{T12} \\ y_{C21} \\ y_{C22} \\ y_{T21} \\ y_{T22} \end{bmatrix} \tag{20}$$

$$
V = \begin{bmatrix}
\nu & c_1 & \sigma_F^2 & \sigma_F^2 & 0 & 0 & 0 & 0 \\
c_1 & \nu & \sigma_F^2 & \sigma_F^2 & 0 & 0 & 0 & 0 \\
\sigma_F^2 & \sigma_F^2 & \nu & c_1 & 0 & 0 & 0 & 0 \\
\sigma_F^2 & \sigma_F^2 & c_1 & \nu & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \nu & c_1 & \sigma_F^2 & \sigma_F^2 \\
0 & 0 & 0 & 0 & c_1 & \nu & \sigma_F^2 & \sigma_F^2 \\
0 & 0 & 0 & 0 & \sigma_F^2 & \sigma_F^2 & \nu & c_1 \\
0 & 0 & 0 & 0 & \sigma_F^2 & \sigma_F^2 & c_1 & \nu
\end{bmatrix} \tag{21}
$$