Kingman's coalescent

Random collision of lineages as go back in time (sans recombination) Collision is faster the smaller the effective population size



Coalescence is faster in small populations

Change of population size and coalescents



time

the changes in population size will produce waves of coalescence



The parameters of the growth curve for N_e can be inferred by likelihood methods as they affect the prior probabilities of those trees that fit the data.

Week 9: Coalescents -p.24/60



Figure from Minin, Bloomquist, and Suchard 2008

BEST Liu and Pearl (2007); Edwards et al. (2007)

- X sequence data
- *G* a genealogy (gene tree with branch lengths)
- S a species tree
- θ demographic parameters
- Λ parameters of molecular sequence evolution

$$\Pr(S, \boldsymbol{\theta} | X) = \frac{\Pr(S, \boldsymbol{\theta}) \Pr(X | S, \boldsymbol{\theta})}{\Pr(X)}$$

=
$$\Pr(S) \Pr(\boldsymbol{\theta}) \int \Pr(X | G) \Pr(G | S, \boldsymbol{\theta}) dG$$

$$\propto \Pr(S) \Pr(\boldsymbol{\theta}) \int \left[\int \Pr(X | G, \boldsymbol{\Lambda}) \Pr(\boldsymbol{\Lambda}) d\boldsymbol{\Lambda} \right] \Pr(G | S, \boldsymbol{\theta}) dG$$

- 1. Generate a collection of gene trees, G, using an approximation of the coalescent prior
- 2. Sample from the distribution of the species trees conditional on the gene trees, G.
- 3. Use "importance weights" to correct the sample for the fact that an approximate prior was used

Generate a collection of gene trees, G, using an approximation of the coalescent prior
 (a) Use a tweaked version of MrBayes to sample N sets of gene trees, G, from

$$\Pr^{\dagger}(\boldsymbol{G}|X) = \frac{\Pr^{\dagger}(\boldsymbol{G})\Pr(X|\boldsymbol{G})}{\Pr^{\dagger}(X)}$$

- (b) $Pr^{\dagger}(G)$ is an approximate prior on gene trees from using a "maximal" species tree.
- 2. Sample from the distribution of the species trees conditional on the gene trees, G.
- 3. Use "importance weights" to correct the sample for the fact that an approximate prior was used

- 1. Generate a collection of gene trees, G, using an approximation of the coalescent prior
- 2. Sample from the distribution of the species trees conditional on the gene trees, G.
 - (a) From each set of gene trees (G_j for $1 \le j \le N$) generate

k species trees using coalescent theory:

$$\Pr(S_i | \boldsymbol{G}_j) = \frac{\Pr(S_i) \Pr(\boldsymbol{G}_j | S_i)}{\Pr(\boldsymbol{G}_j)}$$

3. Use "importance weights" to correct the sample for the fact that an approximate prior was used

- 1. Generate a collection of gene trees, G, using an approximation of the coalescent prior
- 2. Sample from the distribution of the species trees conditional on the gene trees, G.
- 3. Use "importance weights" to correct the sample for the fact that an approximate prior was used
 - (a) Estimate $\widehat{\Pr}(G_j)$ by using the harmonic mean estimator from the MCMC in step 2.
 - (b) Compute a normalization factor

$$\beta = \sum_{j=1}^{N} \frac{\widehat{\Pr}(\boldsymbol{G}_j)}{\Pr(\boldsymbol{G}_j)}$$

(c) Reweight all sampled species trees by

$$rac{\widehat{\Pr}(oldsymbol{G}_j)}{\Pr(oldsymbol{G}_j)}eta$$

- 1. very expensive computationally (long MrBayes runs are needed)
- 2. should correctly deal with the variability in gene tree caused by the coalescent process.

Goal: approximate Pr(S|X)

$$Pr(S|X) \propto Pr(X|S) Pr(S)$$

$$= \int Pr(X|G) Pr(G|S) Pr(S) dG$$

$$= \int \int \int Pr(X|G) Pr(G|S, \theta) Pr(S) dG d\theta$$

$$= \int \int \int \int Pr(X|G, \Lambda) Pr(G|S, \theta) Pr(S) dG d\theta d\Lambda$$

$$\theta = \{N_1, N_2, \dots, \}$$

$$\Lambda = \{\kappa, \pi, \dots\}$$







G





Figure modified from Heled and Drummond 2010















Figure modified from Heled and Drummond 2010



Changing G affects $\Pr(X|G, \mathbf{\Lambda})$ and $\Pr(G|S, \boldsymbol{\theta})$

Another MCMC update to gene tree



Some changes to G are incompatible with S (and will be rejected).



Changing S affects $Pr(G|S, \theta)$, but not $Pr(X|G, \Lambda)$.

Note that the red dots are "flags" for when a lineage enters a new species; the heights are determined by the species tree.

Another MCMC update to species tree



Some changes to S are incompatible with C (and will be rejected).

An MCMC update to the population size



 $N_e \in \boldsymbol{\theta}$, so changing N_e affects $\Pr(G|S, \boldsymbol{\theta})$, but not $\Pr(X|G, \boldsymbol{\Lambda})$.

Multiple gene tree in a species tree w/ variable population size



Figure from modified Heled and Drummond 2010

Similar model to BEST, but *much* more efficient implementation.

Both attempt to sample the posterior distribution of species trees, gene trees, demographic parameter values and mutational parameter values.

Both will be very sensitive to migration, but they represent the state-of-the-art for estimating species trees from gene trees.

- The goal of MSA is to introduce gaps such that residues in the same column are homologous (all residues in the column descended from a residue in their common ancestor).
- The problem is recast as:
 - reward matches (+ scores)
 - penalize rare substitutions (- scores),
 - penalize gaps (- scores),
 - try to find an alignment that maximizes the total score
- pairwise alignment is tractable
- MSA is usually done progressively
- progressive alignment algorithms are heuristic, and do not optimize an evolutionary defensible criterion

- clustal variants are popular, but not very reliable.
- simultaneous inference of MSA and tree is the most defensible (but computationally demanding)
- Promising tools for MSA (roughly in order of computational tractability):
 - 1. Simultaneous MSA + Trees (Handel, BAliPhy, BEAST, AliFritz...)
 - 2. FSA (fast statistical alignment); Infernal (for rRNA); Prank
 - 3. MAFFT, Muscle, ProbCons
- Iterative "meta-solutions" (e.g. SATè) allow MSA uncertainty to be incorporated in tree inference.
- GBlocks (and similar tools) cull ambiguously aligned regions.

human	KRSV
chimp	KRV
orang	KPRV



human	KRSV
chimp	KRV
gorilla	KSV
orang	KPRV

How should we align these sequences?

human	KRSV		human	KRSV
chimp	KR-V	OR	chimp	K-RV
gorilla	KS-V		gorilla	K-SV
orang	KPRV		orang	KPRV

Gap penalties and a substitution matrix imply a score for any alignment. Pairwise alignment involves finding the alignment that maximizes this score.

- substitution matrices assign positive values to matches or similar substitutions (for example Leucine→Isoleucine).
- unlikely substitutions receive negative scores
- gaps are rare and are heavily penalized (given large negative values).

Costs:

						Μ	ato	ch		1							
					Ν	/lis	ma	Itcl	h	0							
						(Gap)		-5							
Alignment:																	
Pongo	V	D	Е	V	G	G	Е	L	G	R	L	F	V	V	Р	Т	Q
Gorilla	V	Е	V	А	G	D	L	G	R	L	L	Ι	V	Y	Р	S	R
Score	1	0	0	0	1	0	0	0	0	0	1	0	1	0	1	0	0

Total score = 5



Total score = 0

BLOSUM 62 Substitution matrix

	Α	R	N	D	C	Q	E	G	Н	I	L	К	M	F	P	S	Т	W	Y	V
Α	4																			
R	-1	5																		
Ν	-2	0	6																	
D	-2	-2	1	6																
С	0	-3	-3	-3	9															
Q	-1	1	0	0	-3	5														
E	-1	0	0	2	-4	2	5													
G	0	-2	0	-1	-3	-2	-2	6												
Н	-2	0	1	-1	-3	0	0	-2	8											
I	-1	-3	-3	-3	-1	-3	-3	-4	-3	4										
L	-1	-2	-3	-4	-1	-2	-3	-4	-3	2	4									
K	-1	2	0	-1	-3	1	1	-2	-1	-3	-2	5								
Μ	-1	-1	-2	-3	-1	0	-2	-3	-2	1	2	-1	5							
F	-2	-3	-3	-3	-2	-3	-3	-3	-1	0	0	-3	0	6						
Р	-1	-2	-2	-1	-3	-1	-1	-2	-2	-3	-3	-1	-2	-4	7					
S	1	-1	1	0	-1	0	0	0	-1	-2	-2	0	-1	-2	-1	4				
Т	0	-1	0	-1	-1	-1	-1	-2	-2	-1	-1	-1	-1	-2	-1	1	5			
W	-3	-3	-4	-4	-2	-2	-3	-2	-2	-3	-2	-3	-1	1	-4	-3	-2	11		
Y	-2	-2	-2	-3	-2	-1	-2	-3	2	-1	-1	-2	-1	3	-3	-2	-2	2	7	
V	0	-3	-3	-3	-1	-2	-2	-3	-3	3	1	-2	1	-1	-2	-2	0	-3	-1	4
	Α	R	N	D	C	Q	E	G	Н	I	L	K	M	F	Р	S	Т	W	Y	V

Scoring an alignment with the BLOSUM 62 matrix

Pongo	V	D	E	V	G	G	E	L	G	R	L	F	V	V	Ρ	Т	Q
Gorilla	V	Е	V	А	G	D	L	G	R	L	L	I	V	Y	Р	S	R
Score	4	2	-2	0	6	-6	-3	-4	-2	-2	4	0	4	-1	7	4	1

The score for the alignment is

$$D_{ij} = \sum_{k} d_{ij}^{(k)}$$

If i indicates *Pongo* and j indicates *Gorilla*

$$D_{ij} = 12$$
If the GP is -8:

Pongo	V	D	E	V	G	G	Е	L	G	R	L	-	F	V	V	Р	Т	Q
Gorilla	V	-	Ε	V	А	G	D	L	G	R	L	L	Ι	V	Y	Р	S	R
Score	4	-8	5	5	0	6	2	4	6	5	4	-8	0	4	-1	7	4	1

By introducing gaps we have improved the score:

 $D_{ij} = 40$

Gaps are penalized more heavily than substitutions to avoid alignments like this:

PongoVDEVGGE-LGRLFVVPTQGorillaVDEVGG-WLGRLFVVPTQ

Because multiple residues are often inserted or deleted at the same time, *affine gap penalties* are often used:

$\mathsf{GP}=\mathsf{GO}+l\mathsf{GE}$

where:

- GP is the gap penalty.
- GO is the "gap-opening penalty"
- GE is the "gap-extension penalty"
- l is the length of the gap

Finding an optimal alignment



Aligning two sequences, each with length = 1









































length Seq $\#~1$	length Seq $\#$ 2	# alignments
1	1	3
2	2	13
3	3	63
4	4	321
5	5	1,683
6	6	8,989
7	7	48,639
8	8	265,729
9	9	1,462,563
:	E	:
17	17	1,425,834,724,419

Needleman-Wunsch algorithm (paraphrased)

- Work from the top left (beginning of both sequences)
- For each cell store the highest score possible for that cell and a "back" pointer to tell point to the previous step in the best path
- When you reach the lower right corner, you know the optimal score and the back pointers tell you the alignment.

The highest-score calculation at each cell only depends on its the cell's three possible previous neighbors.

If one sequence is length N, and the other is length M, then Needleman-Wunsch only takes $\approx 6NM$ calculations.

But there are a **much** larger number of possible alignments.













		V	D	E	V	G	G	E	L	G	R
V	0 ↑	← -5	← -10	← -15	← -20	← -25	← -30	← -35	← -40	← -45	← -50
	-5	4	-1	↔ -6	-11	← -16	← -21	← -26	← -31	← -36	← -41
Е	Ť	\uparrow	~	~				~			
V	-10 ↑	-1	6 ↑	4	← -1 ↖	<i>←</i> -6	← -11	-16	← -21	← -26	← -31
A	-15 ↑	-6 ↑	1 ↑	4	8	← 3	← -2	← -7	← -12	← -17	← -22
G	-20 ↑	-11 ↑	-4 ↑	0 ↑	4 ↑	8	3	← -2	← -7	-12	← -17
D	-25 ↑	-16 ↑	-9	-5	-1 ↑	10 ↑	14	← 9 ×	← 4	-1	-6 ↔
L	-30 ↑	-21 ↑	-10 ↑	-7 ↑	-6	5 ↑	9 ↑	16 ↑	← 11 ×	← 6	← 1
G	-35 ↑	-26 ↑	-15 ↑	-12	-6 ↑	0	4	11 ↑	20 ↑	← 15 [►]	← 10
R	-40 ↑	-31 ↑	-20 ↑	-17	-11 ↑	0 ↑	6 ↑	6	15 ↑	26 ↑	← 21
L	-45 ↑	-36 ↑	-25 ↑	-20 ↑	-16	-5 ↑	1 ↑	6 ↑	10	21 ↑	31 ↑
L	-50 ↑	- 41 ↑	-30 ↑	-25 ↑	-19	-10 ↑	-4 ↑	1 ↑	10	16 ↑	26 ↑
I	-55 ↑	-46 ↑	-35 ↑	-30 ↑	-24	-15 ↑	-9 ↑	-4 ↑	5 ↑	11 ↑	21 ↑
. /	-60	-51	-40	-35	-27	-20	-14	-9	0	6	16



Devised by Feng and Doolittle 1987 and Higgins and Sharp, 1988. An approximate method for producing multiple sequence alignments using a guide tree.

- Perform pairwise alignments to produce a distance matrix
- Produce a guide tree from the distances
- Use the guide tree to specify the ordering used for aligning sequences, closest to furthest.


Sequences of clades become grouped into "profiles" as the algorithm descends the tree. The next youngest internal nodes is selected at each step to create a new profile. Alignment at each step involves

- Sequence-Sequence
- Sequence-Profile
- Profile-Profile





Adding a gap to a profile means that every member of that group of sequences gets a gap at that position of the sequence.

Usually the scores for each edge in the Needleman-Wünsch graph are calculated using a "sum of pairs" scoring system.

clustal W¹ uses weights assigned to each sequence in a profile group to downweight closely related sequences so that they are not overrepresented.

¹Thompson, Higgins, and Gibson. Nuc. Acids. Res. 1994

Pro	file 1		Profile 2								
Seq	weight	AA	Sea	woight	ΔΔ						
taxon A	0.3	V		wergin							
ouxon A	0.0	v	taxon B	0.15	V						
taxon C	0.24	Α	-		•						
	0 10	-	taxon D	0.25	М						
taxon E	0.19	T									

$$D_{P1,P2} = \frac{\sum_{i} \sum_{j} w_{i} w_{j} d_{ij}}{n_{i} n_{j}}$$

$$= \frac{1}{6} [d(V,V) w_{A} w_{B} + d(V,M) w_{A} w_{D} + d(A,V) w_{C} w_{B} \dots$$

$$= \dots d(A,M) w_{C} w_{D} + d(I,V) w_{E} w_{B} + d(I,M) w_{E} w_{D}]$$

$$= \frac{1}{6} (\mathbf{4} \times 0.3 \times 0.15 + \mathbf{1} \times 0.3 \times 0.25 + \mathbf{0} \times 0.24 \times 0.15 \dots$$

$$= \dots -1 \times 0.24 \times 0.25 + \mathbf{3} \times 0.19 \times 0.15 + \mathbf{1} \times 0.19 \times 0.15$$

$$= 1.46225$$

Dealing with alignment ambiguity²

	(a)	X Y 1 2 3 4 5 6 7 8 9	Z 1 1 0 1
	Outgroup Taxon A Taxon B Taxon C Taxon D Taxon E	TAGCACTTAGCACTTAGTGAAGCTAGTGAAGCTAGTGAAGCTAGTGAAGCTAGAAGCTAGAAGCTAGAGCTAGAGC	 C A G
(b) $\frac{x}{1 \ 2 \ 3} \frac{4}{4}$	Y 5 6 7 8 9 1 0	z (C) 1 1 1 2	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
OutgroupTAGATaxonATAGATaxonBTAGTTaxonCTAGT	GCACTCGCACTCGAAGCCGAAGCC	A GOutgroupA GTaxon AA GTaxon BA GTaxon C	TAGAGCACTCAGTAGCACTCAGCAGTAGTGAAGCCAGCTAGTGAAGCCAGTAGTGAAGCCAG

²from M. S. Y. Lee, *TREE*, 2001

	(a)					1	<u>x</u> 2	3	4	5	<u>6</u>	<u>7</u>	8	9	1 0	Z 1 1	1 2								
				Out Tax Tax Tax Tax	kor kor kor kor kor	roup A A B C A D A E	T T T T T	A A A A A	6 6 6 6 6 6 7 8 8	A A T T	G G G Z	C C A A A C	A A A G G	C C G G	T T C C	C C C C C C	A A A A A	G G G G G G G							
		x			\mathbf{Z}										x				2	ζ				Z	
	1	2	3	1 0	1 1	1 2								1	2	3	4	5	6	7	8	9	1 0	1 1	1 2
Outgroup	т	Α	G	C	Α	G	Outgroup					т	Α	G	Α	G	C	Α	C	Т	C	Α	G		
Taxon A	т	Α	G	C	Α	G		1	Гa:	xoı	n .	A		т	Α	G	A	G	C	Α	C	т	C	Α	G
Taxon B	Т	Α	G	C	Α	G		1	Гa:	xoı	n	B		т	Α	G	T	G	Α	Α	G	C	C	Α	G
Taxon C	Т	Α	G	C	Α	G		1	Гa:	xoı	n	C		т	Α	G	Т	G	Α	Α	G	C	C	Α	G
Taxon D	Т	Α	G	C	Α	G		1	Гa:	xoı	n	D		т	Α	G	?	?	?	-	-	-	C	Α	G
Taxon E	\mathbf{T}	Α	G	C	Α	G		1	Гa:	xoı	n	E		т	Α	G	?	?	?	-	-	-	C	Α	G

³from M. S. Y. Lee, *TREE*, 2001

Elision method (Wheeler, 1995) involves simply concatenating matrices.



⁴from M. S. Y. Lee, *TREE*, 2001

Simultaneous tree inference and alignment

- Ideally we would address uncertainty in both types of inference at the same time
- Allows for application of statistical models to improve inference and assessments of reliability
- Just now becoming feasible: POY (Wheeler, Gladstein, Laet, 2002), Handel (Holmes and Bruno, 2001), BAliPhy (Redelings and Suchard, 2005), and BEAST(Lunter *et al.*, 2005, Drummond and Rambaut, 2003). SATe (Liu *et al* 2009; Yu and Holder software).

- Edwards, S. V., Liu, L., and Pearl, D. K. (2007). Highresolution species trees without concatenation. *Proceedings* of the National Academy of Sciences, 104(14):5936–5941.
- Liu, L. and Pearl, D. K. (2007). Species trees from gene trees: reconstruction Bayesian posterior distributions of a species phylogeny using estimated gene tree distributions. *Systematic Biology*, 56(3):504–514.