

In the model jumping move that we covered in class the relevant parts were:

```
elif model_index == 1:
    if move_choice_u < 0.3333333333333333:
        u = random.random()
        sigma_star = -log(1 - u)/sigma_prior_hazard_param
        theta_to_alter[1] = sigma_star
        theta_to_alter[2] = 2 # Jump to model 2
```

and:

```
if move_choice_u < 0.3333333333333333:
    sigma = theta_to_alter[1]
    ln_hastings_ratio = log(sigma_prior_hazard_param) - sigma_prior_hazard_param*sigma
    theta_to_alter[1] = 1.0
    theta_to_alter[2] = 1 # Jump to model 1
```

One of the first things to realize is that the 1 or 2 variable in the index 2 of `theta_to_alter` is not really a parameter, it is just a flag that tells us what model we are using.

So the math here is that the jump from model #1 to model 2 is:

$$u \sim U(0, 1)$$
$$\sigma^* = \frac{-\ln(1 - u)}{\lambda_\sigma}$$

where λ_σ is the hazard parameter of the exponential distribution that we are using as a proposal distribution (by coincidence, I chose this same distribution as my prior for σ , but that is not required for the move to work). So in there is really only 1 parameter (and one random variable) to analyze here.

Because the density of u is 1.0, the Hasting's ratio is just the absolute value of the determinant of the Jacobian. And the Jacobian is just a 1×1 matrix, so the determinant is the only element of the matrix namely:

$$\frac{\partial \sigma^*}{\partial u} = \frac{1}{(1 - u)\lambda_\sigma}$$

Just rephrasing that result to get rid of the reference to u we get:

$$\sigma^* = \frac{-\ln(1 - u)}{\lambda_\sigma}$$
$$\sigma^* \lambda_\sigma = -\ln(1 - u)$$
$$e^{-\sigma^* \lambda_\sigma} = 1 - u$$
$$J = \frac{\partial \sigma^*}{\partial u} = \frac{1}{(1 - u)\lambda_\sigma}$$
$$= \left(\frac{1}{\lambda_\sigma}\right) \left(e^{\sigma^* \lambda_\sigma}\right)$$
$$\ln J = -\ln(\lambda_\sigma) + \sigma^* \lambda_\sigma$$

This is how I derived the log Hasting's ratio in the code:

```
ln_hastings_ratio = -log(sigma_prior_hazard_param) + sigma_prior_hazard_param*sigma_star
```