

Numerical optimization – minimizing a function by evaluating it at many trial points.

Main points:

1. optimizers can fail to find the global optimum:
  - (a) multiple modes are a problem.
  - (b) result is often starting point dependent.
2. limited precision in computers → rounding error, which complicates termination criteria.

## Numerical optimization – practical recommendations.

1. Try multiple starting points.
2. Try multiple optimization algorithms.
3. Don't skimp on optimization in your parametric bootstrapping (or at least make sure that the search for the global optimum is a good search).
4. Reparameterization can help
5. Using derivatives from finite differences can be surprisingly effective – consider BFGS even if you can't calculate the gradient.

You suspect that a population of big horn sheep are made up of two classes of males based on their sparring ability: Strong and Weak. The proportion of strong individuals is unknown.

### **Experiment:**

- You randomly select 10 pairs of males from a large population.
- For each pair you randomly assign one of them the ID 0 and the other the ID 1.
- You record the # of winner from 2 contests.

### **Model:**

- If two individuals within the same class fight, you expect either outcome to be equally likely.
- If a Strong is paired against a Weak then you expect that the probability that the stronger one wins with some probability,  $w$ .
- $w$  is assumed to be the same for every pairing of Strong versus Weak and the same for every bout within such a pairing.

Pair #	winner	
	bout 1	bout 2
1	1	1
2	1	0
3	0	1
4	1	1
5	0	0
6	0	1
7	1	1
8	0	0
9	1	0
10	1	1

What can we say about  $w$ ?

Pair #	winner	
	bout 1	bout 2
1	1	1
2	1	0
3	0	1
4	1	1
5	0	0
6	0	1
7	1	1
8	0	0
9	1	0
10	1	1

$X$

$$x_1 = 1 \quad x_{11} = 1$$

$$x_2 = 1 \quad x_{12} = 0$$

$$x_3 = 0 \quad x_{13} = 1$$

$$x_4 = 1 \quad x_{14} = 1$$

$$x_5 = 0 \quad x_{15} = 0$$

$$x_6 = 0 \quad x_{16} = 1$$

$$x_7 = 1 \quad x_{17} = 1$$

$$x_8 = 0 \quad x_{18} = 0$$

$$x_9 = 1 \quad x_{19} = 0$$

$$x_{10} = 1 \quad x_{20} = 1$$

$$L(w, p_S) = \prod_{i=1}^{20} \mathbb{P}(x_i | w, p_S)$$

Pair #	winner	
	bout 1	bout 2
1	1	1
2	1	0
3	0	1
4	1	1
5	0	0
6	0	1
7	1	1
8	0	0
9	1	0
10	1	1

$X$

$$\begin{array}{ll}
x_1 = 1 & x_{11} = 1 \\
x_2 = 1 & x_{12} = 0 \\
x_3 = 0 & x_{13} = 1 \\
x_4 = 1 & x_{14} = 1 \\
x_5 = 0 & x_{15} = 0 \\
x_6 = 0 & x_{16} = 1 \\
x_7 = 1 & x_{17} = 1 \\
x_8 = 0 & x_{18} = 0 \\
x_9 = 1 & x_{19} = 0 \\
x_{10} = 1 & x_{20} = 1
\end{array}$$

$$\mathbb{P}(x_{11} = 1 | x_1 = 1, w, p_S) \neq \mathbb{P}(x_{11} = 1)$$

$$L(w, p_S) = \prod_{i=1}^{10} \mathbb{P}(x_i | w, p_S) \mathbb{P}(x_{10+i} | x_i, w, p_S)$$

Pair #	winner	
	bout 1	bout 2
1	1	1
2	1	0
3	0	1
4	1	1
5	0	0
6	0	1
7	1	1
8	0	0
9	1	0
10	1	1

$Z = (\# \text{ won by 0, } \# \text{ won by 1})$

$$z_1 = (0, 2)$$

$$z_2 = (1, 1)$$

$$z_3 = (1, 1)$$

$$z_4 = (0, 2)$$

$$z_5 = (2, 0)$$

$$z_6 = (1, 1)$$

$$z_7 = (0, 2)$$

$$z_8 = (2, 0)$$

$$z_9 = (1, 1)$$

$$z_{10} = (0, 2)$$

$$L(w, p_S) = \prod_{i=1}^{10} \mathbb{P}(z_i | w, p_S)$$

Let  $m$  be the mismatch status:

$$m \in \left\{ \begin{array}{l} 0 \text{ (even)} \\ 1 \text{ (mismatched)} \end{array} \right\}$$

$$L(w, p_S) = \prod_{i=1}^{10} [\mathbb{P}(z_i|w, m = 0)\mathbb{P}(m = 0|p_S) + \mathbb{P}(z_i|w, m = 1)\mathbb{P}(m = 1|p_S)]$$